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COMMENT

Comment on 'Towards gravitationally assisted negative refraction of light by vacuum'

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Abstract
A recent paper in this journal claimed that approximately flat regions of spacetime remote from matter may support electromagnetic waves for which the phase velocity 3-vector is oppositely directed to the power flow 3-vector. We here show that this cannot occur in regions of spacetime for which the electromagnetic energy density is positive.

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In [1] it was claimed that matter-free regions of spacetime may support electromagnetic waves for which the phase velocity 3-vector is oppositely directed to the power flow 3-vector. This was the definition of the so-called 'negative refraction' adopted in [2]. This comment demonstrates that this claim is incompatible with the positivity of electromagnetic energy density.

In complex notation, the stress energy tensor associated with the electromagnetic field is given by

$$T_{\alpha\beta} = \frac{1}{8\pi}\left[F_{\alpha}{}^{\gamma}(F_{\beta\gamma})^* - \frac{1}{4}g_{\alpha\beta}F_{\mu\nu}(F^{\mu\nu})^*\right], \tag{1}$$

where taking the real part is understood, and the units are Gaussian with $c = 1$. The electromagnetic field tensor may be written in terms of the vector potential via

$$F_{\alpha\beta} = \nabla_{\alpha}A_{\beta} - \nabla_{\beta}A_{\alpha}. \tag{2}$$

Maxwell's equations are

$$\nabla_{\beta}F^{\alpha\beta} = 4\pi J^{\alpha}, \tag{3}$$

and

$$\nabla_{[\gamma}F_{\alpha\beta]} = 0. \tag{4}$$

In terms of the vector potential (4) is satisfied automatically, whilst under the Lorentz gauge ($\nabla_\alpha A^\alpha = 0$) equation (3) becomes

$$\nabla^\alpha \nabla_\alpha A^\beta - R_\delta{}^\beta A^\delta = -4\pi J^\beta, \tag{5}$$

where $R_\delta{}^\beta$ is the Ricci tensor. In vacuum, neglecting any curvature induced by the field, both $R_\delta{}^\beta$ and J^β vanish leaving

$$\nabla^\alpha \nabla_\alpha A^\beta = 0. \tag{6}$$

Plane wave solutions of Maxwell's equations may be written as

$$A^\alpha = C^\alpha \mathrm{e}^{\mathrm{i}K_\mu X^\mu}, \tag{7}$$

where C^α is a covariantly constant 4-vector amplitude[1] and K_μ are the covariant components of the 4-wavevector, $(-\omega, \mathbf{k})$. We take $\omega > 0$. Negative refraction is said to occur whenever k_j is oppositely signed to $-T_{0j}$. Consider (as in [1]) 'a small region of space at a sufficiently remote location from the centre of mass', that the metric can be considered uniform. Can such a spacetime region support 'negative refraction' as defined above? Since in the considered region all covariant derivatives may be replaced by ordinary derivatives, we have from equations (2), (6), (7) and the Lorentz gauge $\left(A^\mu{}_{,\mu} = 0\right)$ that

$$F_{\mu\nu} = \mathrm{i}(K_\mu A_\nu - K_\nu A_\mu), \tag{8}$$

$$K_\mu K^\mu = 0, \tag{9}$$

$$K^\mu A_\mu = 0. \tag{10}$$

Using equations (1), (8), (9) and (10) it is straightforward to show that in this case

$$T_{\alpha\beta} = \frac{1}{8\pi} A^\mu (A_\mu)^* K_\alpha K_\beta, \tag{11}$$

and hence that

$$-T_{0j} = \frac{1}{8\pi} A^\mu (A_\mu)^* \omega k_j. \tag{12}$$

Now since $A^\mu (A_\mu)^* > 0$ (provided $T_{00} > 0$), then k_j must take the same sign as $(-T_{0j})$, and the negative refraction condition cannot be fulfilled. Within the geometric optics approximation, the above argument is also valid with covariant derivatives replacing ordinary derivatives, and is thus valid for curved spacetimes also. We conclude that regions of spacetime in which the electromagnetic energy density is positive cannot support 'negative refraction'.

References

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[3] Wald R M 1984 *General Relativity* (Chicago, IL: University of Chicago Press) p 71

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[1] In flat spacetimes global solutions to Maxwell's equations exist for which the amplitude C^α is covariantly constant over all spacetime. In curved spacetime it is often possible to construct solutions in which C^α varies slowly—the so-called 'geometric optics approximation' [3, 4].